St Mary's Church of England Primary School

Fowler Road, Islington, London N1 2EP

'Believe and Achieve'

Maths Calculation Policy ~ Addition and Subtraction

- This policy contains the key pencil and paper procedures that are to be taught throughout the school.

- It has been written to ensure consistency and progression across the school.

- Although the main focus of this policy is on pencil and paper procedures it is important to recognise the key role of number facts and mental calculation. They should be seen as complementary to written methods and not as separate from it. The ability to calculate mentally lies at the heart of numeracy. In every written method there is an element of mental processing.

- Written recording both helps pupils to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

- In setting out this policy the intention is that all teachers develop an understanding of what progression in calculation looks like. Progression in calculation is a developmental skill that should be taught when the child is ready. Children's advancement in calculation should be at an appropriate time for their ability, which may not meet national expectations for their age. Thus central to the effective implementation of the policy is effective and rigorous day-to-day assessment which allows teachers to determine when a pupil should move on to the next stage within the policy.

- The long-term aim is for children to be able to select an efficient method of their choice that is appropriate for a given task.

End of Year Expectations

The table below shows the end of year expectations for each of the four operations: Please note these are average expectations; some children will be operating above and below age related expectations.

Addition		*NC expectations*
Year 1	Unlabelled number line to bridge through ten -Method 1b	*Add two numbers to 20* *Add 3 1-digit numbers*
Year 2	Unlabelled number line to add TU to TU - Method 1b	*Add two 2-digit numbers* *Use column without carrying*
Year 3	Unlabelled number line to add HTU to HTU - Method 1b Column method to add HTU to HTU (1Carry) - Method 2	*Add two 3-digits numbers* *Use column method*
Year 4	Unlabelled number line to add decimals - Method 1b Column method to add THTU to THTU - Method 2	*Add two 4-digit numbers*
Year 5	Column method to add decimals to three places - Method 2	*Add two 5-digit numbers* *Add numbers- 3 decimal places*
Year 6	Column method to add integers and decimals - Method 2	*Add negative integers*

Subtraction		*NC expectations*
Year 1	Unlabelled number line to subtract two one digit numbers - Method 1b	*Subtract two numbers to 20*
Year 2	Unlabelled number line to subtract TU from TU - Method 1b	*Subtract two 2-digits to 20* *Use column without borrowing*
Year 3	Counting on to subtract TU from TU & find missing numbers - Method 2 Unlabelled number line to subtract HTU from HTU - Method 1b Column method to subtract HTU - HTU (exchange H for T) - Method 3	*Subtract two 3-digit numbers* *Use column method*
Year 4	Counting on to subtract HTU from HTU & find missing numbers - Method 2 Column method to subtract THTU from THTU(noughts in the first number) - Method 3	*Subtract two 4-digit numbers*
Year 5	Counting on to subtract decimals - Method 2 Column method to subtract decimals to two places - Method 3	*Subtract two 5-digit numbers* *Subtract no's - 3 decimal places*
Year 6	Column method to subtract integers and decimals - Method 3	*Subtract negative integers*

Mu**ltiplication**		*NC expectations*
Year 1	Labelled number line to multiply one digit numbers by 2 - Method 2a	*Write (x)symbol in equations* *Use concrete objects to solve*
Year 2	Informal jottings to multiply TU (below 20) by 2 and 5 - Method 3	*Use (x) symbol in equations*
Year 3	Grid method to multiply basic TU by U – Method 4	*Solve 2-digit x 1-digit numbers*
Year 4	Grid method to multiply HTU by U – Method 4	*Solve 2-digit and 3-digit numbers x 1-digit number*
Year 5	Short multiplication to multiply HTU by U - Method 5 Grid method to multiply TU by TU – Method 4	*Solve 4-digit x 1 & 2-digit no.s, include long multiplication* *X numbers by 10, 100 &1000*
Year 6	Short multiplication to multiply HTU by U and U.t by U - Method 5 Grid method to multiply HTU by TU - Method 4	*Solve 4-digits x 2-digit whole number using long multiplication* *X numbers up to three decimal place by 10, 100 & 1000* *X numbers with up to two decimal places by 1-digit and 2-digit numbers*

Division		*NC expectations*
Year 1	Cups to explore the basic concept of grouping - Method 1	*Write (÷) symbol in equations* *Use concrete objects to solve*
Year 2	Informal method (tally) to work out division using grouping - Method 2	*Use (÷) symbol in equations* *(grouping & sharing - halving)*
Year 3	Bus stop method to divide TU by U with no reminders or carrying-Method 3	*Solve ÷ equations* *division within x tables*
Year 4	Bus stop method to divide TU by U with remainders and carrying - Method 3	*Solve 2-digit and 3-digit numbers ÷ 1-digit numbers* *Divide 2-digit number by 10 and 100*
Year 5	Bus stop method to divide HTU by U zeros in the quotient - Method 3	*Solve 4 digits ÷1-digit numbers* *÷ numbers by 10, 100 &1000*
Year 6	Bus stop method to divide HTU by U - Method 3 Short division to divide HTU by TU - Method 4	*Solve 4-digits ÷2-digit numbers using long division* *÷ numbers up to three decimal place by 10, 100 & 1000* *÷ numbers with up to two decimal places by 1-digit & 2-digit numbers*

Written Methods for Addition

The progression in strategies outlined here is vitally important for developing children's conceptual understanding of addition, without which children's mathematics cannot move beyond a basic level.

NB: It is important that children's **mental methods of calculation** and **number facts** are practised and secured alongside their learning and use of an efficient written method for addition.

Calculations should be written on either side of the equals sign so that the sign is not just interpreted as 'the answer'.

Keep referring back to the big picture – addition being the inverse of subtraction

Pre-skills to written methods

Explanation and Steps	Example	Progression of equations
Step 1: Ensure children understand the equality principle (so that the equals sign is not just interpreted as 'the answer').		
Step 2: Count the total number of holes in both pieces of numicon. **Step 3:** Count on in ones from the largest numicon piece.	Addition up to 20  3 + 1 = 4	U + U = is less than 10 U + U = 10 TU + U = less than 20 TU + U = 20 10 + 10 = 20 U + U + U = less than or equal to 20
The number track provides an introduction to the labeled number line (pre-cursor of written methods). **Step 4**: Addition up to 20 Children start at the largest number then count on in ones to find the answer.	Addition up to 20 3 + 2 = 5 	

Things to remember

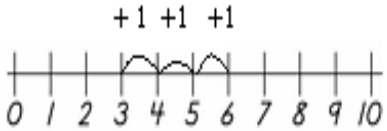
Children need exposure to a variety of different counting apparatus and tangible objects to count and add together at this stage

Method 1a: Labelled number line

The progression in strategies outlined here is vitally important for developing children's conceptual understanding of addition, without which children's mathematics cannot move beyond a basic level.

NB:

- It is important that children's **mental methods of calculation** and **number facts** are practised and secured alongside their learning and use of an efficient written method for addition.
- Calculations should be written on either side of the equals sign so that the sign is not just interpreted as 'the answer'.
- Keep referring back to the big picture – addition being the inverse of subtraction

Explanation and Steps	Example	Progression of equations
The labelled number line provides an introduction to written addition and can be used when children are still insecure with counting. **Step 1:** Addition up to 20 Children start at the largest number then count on in ones to find the answer.	Addition up to 20 3 + 3 = 6  	U + U = less than 10 U + U = 10 U + U = less than 20 TU + U = less than 20 TU + U = 20 10 + 10 = 20 U + U + U = less than or equal to 20

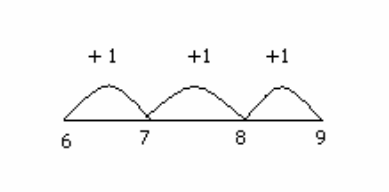
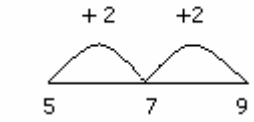
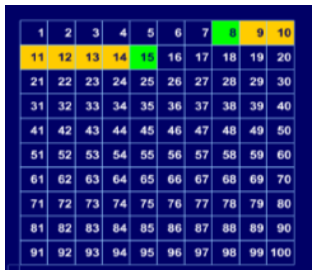
Things to remember

1) It is important that children move to the empty number line as soon as they can reliably count from 0-10 **and** understand the concept of the number line. This is because the labelled line doesn't require mental counting on at each jump; something that is required when adding with the blank number line and, most importantly, mentally.

Method 1b: Unlabelled number line

NB:

- It is important that children's **mental methods of calculation** and **number facts** are practised and secured alongside their learning and use of an efficient written method for addition.
- Calculations should be written on either side of the equals sign so that the sign is not just interpreted as 'the answer'.
- Keep referring back to the big picture – addition being the inverse of subtraction

Explanation and Steps	Example	Progression of equations
The unlabelled number line should be taught using the following steps with teachers only moving on when children have a secure understanding of that step: **Step 1: Adding units** Jumps of 1 -The largest number is placed at the start of the line, children then count on in ones according to the second number. The total after each jump should be marked down. **Taking larger steps:** If children have the ability to make larger jumps then when they are secure with jumps of 1 they should be encouraged to do so. Children should be encouraged to decide on the size of the steps themselves. Some children will however find this difficult; it is fine for them to stick to counting up in 1s. *It is crucial that children have exposure to jumping on using a 100 square while simultaneously using the unlabelled number-line. It supports children when labelling their number-line.	**Jumps of 1** 6 +3 = 9  **Taking larger steps** 5 + 4 = 9   8 + 7 = 15	U + U = more than 10 TU + U = not bridging 20 TU + U = 20 U + U + U = less than or equal to 20 TU + U = bridging 20 TU + U + U = bridging 20

Step 2: Adding multiples of 10 to one- and two-digit numbers

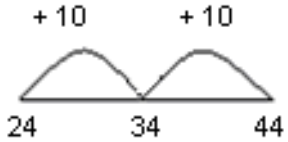
Similar to step 1 but this time children should count on in tens.

To avoid confusion later on please make sure that the multiple of ten is smaller than the number it is being added to. This is because when using a number line to add children are told to put the biggest number at the start of the line.

Taking larger steps: Again, those children with the knowledge and understanding to do so should be encouraged to take jumps of more than 10. Some children will however find this difficult; it is fine for them to stick to counting up in 10s.

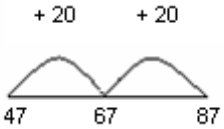
Jumps of 10

24 + 20 =44



Taking larger steps

47 + 40 = 87



TU + multiples of 10 = less than 100

TU + multiples of 10 = more than 100

Step 3: TU + TU

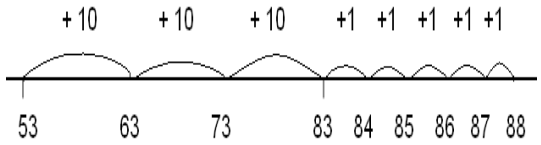
Children should first add the tens and then add the units.

Taking larger steps: Again, those children with the knowledge and understanding to do so should be encouraged to take larger jumps. It is fine if children have not progressed to making larger jumps, they can continue to jump in 10s and 1s respectively.

*It is crucial that children have exposure to jumping on using a 100 square while simultaneously using the unlabelled number-line. It supports children when labelling their number-line.

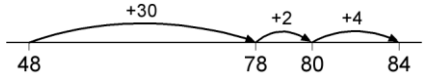
Jumps of 10 and 1

53 + 35 = 88



Taking larger steps

48 + 36 = 84



48 + 36 = 84

TU + TU = not bridging the next 10 (no carrying)

TU + TU = bridging the next 10 (with carrying)

HTU + TU = not bridging the next 10 (no carrying)

HTU + TU = bridging the next 10 (with carrying)

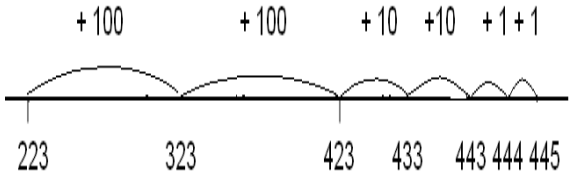
Step 4: HTU + TU & HTU + HTU

Same as above, however this time also using hundreds. Children need to first add the hundreds, then the tens, then the units.

Taking larger steps: Again, those children with the knowledge and understanding to do so should be encouraged to take larger jumps. It is fine if children have not progressed to making larger jumps, they can continue to jump in 100s, 10s and 1s respectively.

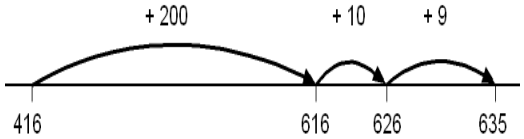
Jumps of 100, 10 and 1

223 + 222 = 445



Taking larger jumps

416 + 219 = 635



HTU + HTU = not bridging the next 10 (no carrying)

HTU + HTU = bridging the next 10 (with carrying)

Things to remember

1. Children need to be able to partition numbers into HTU and, if they are taking larger steps, in ways other than into tens and ones. You will therefore need to ensure that they can do this before attempting the method.
2. Children need to be able to count in tens and hundreds from any given number so this needs to be taught explicitly and regularly practised: whole class chanting is especially effective.
3. The largest number must go first on the number line.
4. The smallest number has to be added in descending order i.e. first hundreds, then tens then units.
5. Ensure at each stage that children are given the opportunity to **add three numbers**.

Method 2: Column Addition

The column method should be taught in the following stages with teachers only moving on when children have a secure understanding of that stage.

NB:

- It is important that children's **mental methods of calculation** and **number facts** are practised and secured alongside their learning and use of an efficient written method for addition.
- Calculations should be written on either side of the equals sign so that the sign is not just interpreted as 'the answer'.
- Keep referring back to the big picture – addition is the inverse of subtraction and can be done in any order (the commutative law)

Explanation and Steps	Example	Progression of equations
Step 1 TU + TU, HTU + TU and HTU +HTU with no carrying	**TU + TU** 32 + 26 58 32 (30 + 2) + 26 (20 + 6) 58 (50 + 8) Teachers may wish to **introduce** column addition using partitioning in this way as it is a sensible mental process for adding the numbers	**No carrying** TU +TU (no carrying) HTU + TU (no carrying) HTU + HTU (no carrying)
Step 2 TU + TU, HTU + TU and HTU + HTU carrying once	**TU + TU** 34 + 29 63 1	**1 carry** TU +TU (1 carry) HTU + TU (1 carry) HTU +HTU (1 carry)

Step 3 TU + TU, HTU + TU and HTU + HTU carrying twice	**HTU + TU** 258 + 87 345 1 1 **HTU + HTU** 366 +458 824 1 1	**2 carries** TU +TU (2 carries) HTU + TU (2 carries) HTU + HTU (2 carries)
Step 4 THTU + HTU and THTU + THTU including a variation of carries	**THTU + HTU** 2353 +868 3221 1 11	**Variation of carries** HTU + HTU THTU + THTU
Step 5 Decimals of the same length including a variation of carries	**U.th + U.th** 7.53 + 1.63 9.16 1	**Decimals of same length & variation of carries** U.t + U.t U.th + U.th
Step 6 Integers and decimals of varying lengths including a variation of carries	**Integers and decimals** 345.00 + 001.63 346.63	**Integers and decimals of varying lengths including a variation of carries** U + U.t TU + U.th Variation

Things to remember

1) Make deliberate connections between informal methods such as number lines and partitioning and formal column procedures.

2) Ensure at each stage that children are given the opportunity to **add three numbers**.

3) Even when they have moved onto the next stage children should still be given opportunities to rehearse and consolidate previous learning.

4) When carrying digits they should be placed underneath, not above.

5) When adding integers and decimals children's accuracy can be improved by getting them to add a decimal point to the integer and make the numbers the same length by including 0s.

6) All children need to be using one digit per square in their books. If children are consistently doing this it makes lining up the digits far easier and helps avoid sloppy mistakes.

7) Ensure that the children use place value vocabulary to explain the process. e.g. 53+ 36= 89

   3 **units** plus 6 **units** equals 9 **units**, put the 9 **units** in the how many box under the units column. 5 **tens** plus 3 **tens** equals 8 **tens**. Put the 8 **tens** in the how many box under the tens column. Question where the 0 goes. Emphasise it is behind the 9 - linking back to partitioning and place value work.

   Ensure that the above process is talked through at each stage of addition.

Time and negative numbers

Children should always use a number line when adding time and when calculating with negative numbers.

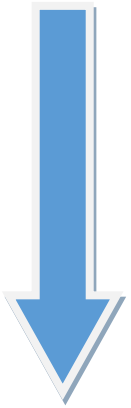
Written Methods for Subtraction

While providing children with a strategy to answer subtraction questions and problems the progression in methods below is vitally important for developing children's conceptual understanding of subtraction without which children's mathematics cannot move beyond a basic level.

NB:

- It is important that children's **mental methods of calculation** and **number facts** are practised and secured alongside their learning and use of an efficient written method for subtraction.
- Keep referring back to the big picture – addition being the inverse of subtraction

Pre-skills to written methods

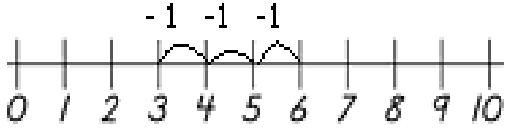
Explanation and Steps	Example	Progression of equations
Step 1: Ensure children understand the equality principle (so that the equals sign is not just interpreted as 'the answer').		
Step 2: Start with the first number. Take away the second number.	Subtraction from 20 7 – 2 = 5 	U – U 10 – U TU – U = minuend less than 20 20 – U 20 – 10 
The number track provides an introduction to the labeled number line (pre-cursor of written methods). **Step 3**: Subtraction from 20 Children start at the first number then count back in ones to find the answer.	Subtraction from 20 5 – 2 = 3 	
Things to remember Children need exposure to a range of counting apparatus and tangible objects to count and subtract at this stage		

Method 1a: Labelled number line

While providing children with a strategy to answer subtraction questions and problems the progression in methods below is vitally important for developing children's conceptual understanding of subtraction without which children's mathematics cannot move beyond a basic level.

NB:

- It is important that children's **mental methods of calculation** and **number facts** are practised and secured alongside their learning and use of an efficient written method for subtraction.
- Keep referring back to the big picture – addition being the inverse of subtraction

Explanation and Steps	Example	Progression of equations
The labelled number line provides an introduction to written subtraction and can be used when children are still insecure with counting. **Step 1-** Subtraction U-U Children start at the first number then count back in ones to find the answer.	Subtraction U - U 6 - 3 = 3  	U – U 10 – U TU – U = minuend less than 20 20 – U 20 – 10

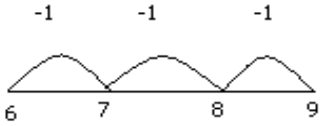
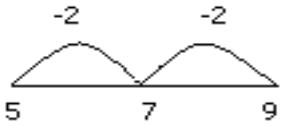
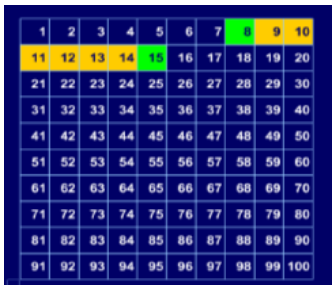
Things to remember

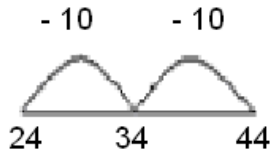
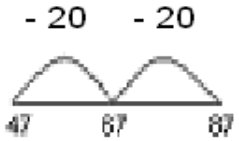
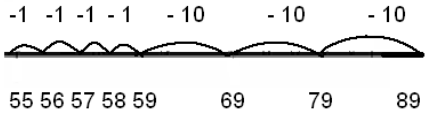
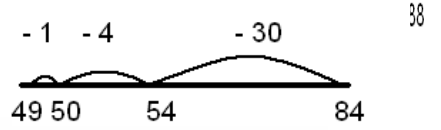
1) It is important that children move to the empty number line as soon as they can reliably count backwards from 10-0 **and** understand the concept of the number line. The reason being that the labelled line doesn't require mental counting back at each jump; something that is required when adding with the blank number line and, most importantly, mentally.

Method 1b: Unlabelled number line

NB:

- It is important that children's **mental methods of calculation** and **number facts** are practised and secured alongside their learning and use of an efficient written method for subtraction.
- Keep referring back to the big picture – addition being the inverse of subtraction

Explanation and Steps	Example	Progression of equations
The unlabelled number line should be taught using the following steps with teachers only moving on when children have a secure understanding of that step: **Step 1: Subtracting units** Jumps of 1 -The first number is placed at the end of the line, children then count back in ones according to the second number. The total after each jump should be marked down. **Taking larger steps:** If children have the ability to make larger jumps then when they are secure with jumps of 1 they should be encouraged to do so. Children should be encouraged to decide on the size of the steps themselves. Some children will however find this difficult; it is fine for them to stick to counting back in 1s. *It is crucial that children have exposure to jumping back in ones using a 100 square while simultaneously using the unlabelled number-line. It supports children when labelling their number-line.	**Jumps of 1** 9 - 3 = 6  **Taking larger steps** 9 - 4 = 5   **15 – 7 = 8**	U – U 10 – U TU – U = minuend less than 20 20 – U 20 – 10

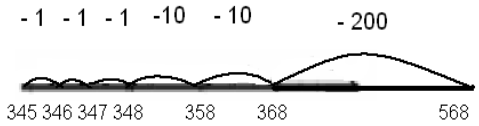
Step 2: Subtracting multiples of 10 from two-digit numbers Similar to step 1 but this time children should count back in tens. **Taking larger steps:** Again, those children with the knowledge and understanding to do so should be encouraged to take jumps of more than 10	**Jumps of 10** 44 - 20 = 24  **Taking larger steps** 87 - 40 = 47  	Multiple of 10 – 10 TU – 10 Multiple of 10 – Multiple of 10 TU – Multiple of 10
Step 3: TU - TU Children should first subtract the tens before then subtracting the units. **Taking larger steps:** If children have not progressed to making larger jumps then it is fine for them to do this in jumps of 10 and 1 respectively.	**Jumps of 10 and 1** 89 - 34 = 55  **Taking larger steps** 84 – 35 = 49  	TU – TU

Step 4: HTU - HTU

Same as above, however this time also using hundreds. Children need to first subtract the hundreds, then the tens, then the units.

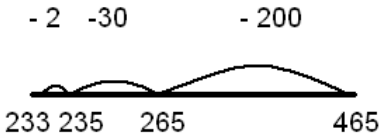
Jumps of 100, 10 and 1

568 - 223 = 345



Taking larger jumps

465 - 232 = 233



HTU – 10 and multiples of 10 without crossing 100

HTU – 10 and multiples of 10 crossing 100

HTU – TU

HTU – HTU (multiples of 10 and 5)

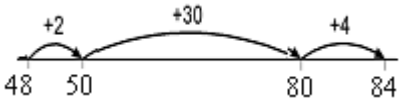
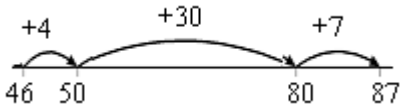
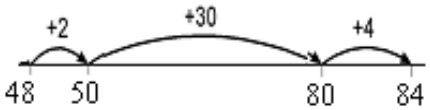
HTU – HTU

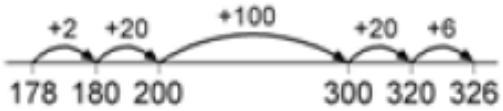
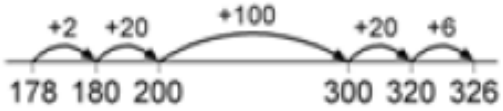
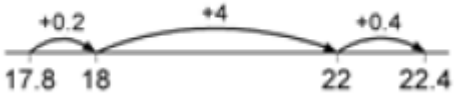
Things to remember

1. Children need to be able to partition numbers into HTU and, if they are taking larger steps, in ways other than into tens and ones. You will therefore need to ensure that they can do this before attempting the method.
2. Children need to be able to count back in units, tens and hundreds from any given number so this needs to be taught explicitly and regularly practised: whole class chanting is especially effective.
3. The first number (the minuend) must go at the end of the number line.
4. The numbers have to be subtracted in descending order i.e. first hundreds, then tens then units.

Method 2: Counting on

When children are confident with both the **concept** and **method** of subtraction by counting back using the number line they should be introduced to this method. It encourages children to see the bigger picture in maths and how closely addition and subtraction are linked.

Explanation and Steps	Example	Progression of equations
Step 1: TU – TU by counting on **Units greater in the first number:** The second number is placed at the start of the line and the first number at the end. Start by jumping in units to the nearest ten. Once this has been done jump in tens to the multiple of ten before finally jumping in units to the final number. The answer is then the total of all the jumps. **Units greater in the second number:** Some children may find these more difficult as the jumps of units may total more than 10. The most able children should be able to do this by counting in tens first and then adding the units but for the least able stick to jumping to the nearest ten first.	**Units greater in second number** 84 – 48 = 36  **Units greater in the first number** 87 – 46 = 41  	TU – TU (units greater in the second number) TU – TU (units greater in the first number)
Step 2: Finding missing numbers TU – □ = TU The difference (answer) is placed at the start of the line and the minuend (first number) at the end. Start by jumping in units to the nearest ten. Once this has been done jump in tens to the multiple of ten before finally jumping in units to the final number. The answer is then the total of all the jumps.	**Finding missing numbers** 84 – □ = 48  	TU – □ = TU

Step 3: HTU – HTU The same as above but this time they first jump to the nearest ten then the nearest hundred.	**HTU - HTU** 326 – 178 = 148  	HTU – HTU
Step 4: Finding missing numbers HTU – ☐ = HTU As above	**HTU - HTU** 326 – 148 = 178  	HTU - ☐ = HTU
Step 5: Decimals As above but this time jump to the nearest integer first.	**Decimals** 22.4 – 17.8 = 4.6  	Decimals

Things to remember

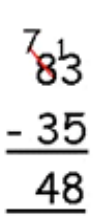
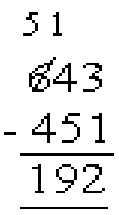
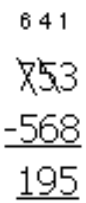
This method is far more efficient when the two numbers being subtracted are close together. Once they have learnt the method give the children lots of practice at looking at a calculation and deciding whether to count forwards or backwards.

Method 3: Column Subtraction

The column method should be taught in the following stages with teachers only moving on when children have a secure understanding of that stage.

NB:

- It is important that children's **mental methods of calculation** and **number facts** are practised and secured alongside their learning and use of an efficient written method for subtraction.
- Keep referring back to the big picture – addition is the inverse of subtraction and can be done in any order (the commutative law)

Explanation and Steps	Example	Progression of equations
Step 1: TU –TU, HTU – TU and HTU – HTU with no adjustments	TU – TU 89 - 24 65	TU – TU (no exchange) HTU – TU (no exchange) HTU – HTU (no exchange)
Step 2: One exchange - Exchange T for U and H for T	**T for U**  **H for T** 	Exchange T for U Exchange H for T
Step 3: Two exchanges	**HTU –HTU** 	TU –TU (2 exchanges) HTU – TU (2 exchanges) HTU – HTU (2 exchanges)

Step 4: Noughts in the first number	**THTU -THTU** 9 9 6 ⁄ 1 7 0 0 3 - 2105 4898	HTU – HTU (noughts in the first number) THTU – THTU (noughts in the first number)
Step 5: Decimals of the same length including a variation of exchange	**TU.th -TU.th** 8 1 79,73 - 21.82 57.91	U.T – U.T U.th – U.th TU.th – TU.th
Step 6: Integers and decimals of varying lengths including a variation of exchange	**Integers and decimals** 346.63 - 001.42 345.21	U – U.t TU – U.th Variation

Things to remember

1) Even when they have moved onto the next stage children should still be given opportunities to rehearse and consolidate these skills.
2) At each stage of the process the value of the digits being subtracted needs to be made explicit to the children. E.g. 6 hundredths subtract 3 hundredths equals 3 hundredths.
1) When subtracting integers and decimals children's accuracy can be improved by getting them to add a decimal point to the integer and make the numbers the same length by including 0s.
2) All children need to be using one digit per square in their books. If children are consistently doing this it makes lining up the digits far easier and helps avoid sloppy mistakes.

Time and negative numbers

Children should always use a number line when subtracting time and when calculating with negative numbers.