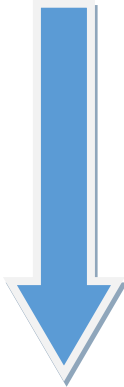Written methods for multiplication

When providing children with a strategy to answer multiplication questions and problems the progression in methods below is vitally important for developing children's conceptual understanding of multiplication without which children's mathematics cannot move beyond a basic level.
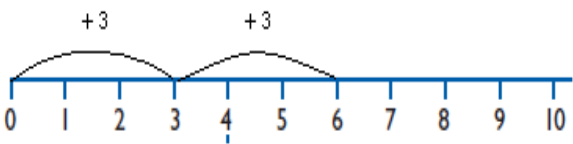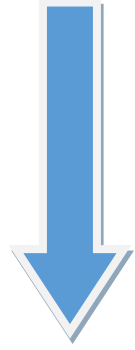
**NB:**

- It is important that children's **mental methods of calculation** and **number facts (times tables)** are practised and secured alongside their learning and use of an efficient written method for multiplication.
- Keep referring back to the big picture – multiplication is the inverse of division, multiplication is repeated addition (adding groups), division is repeated subtraction (subtracting groups)

# Method 1: Introducing the concept of multiplication

| Explanation and Steps | Example | Progression of equations |
|---|---|---|
| Numicon<br>Numicon is used extensively in the foundation stage with a focus on addition. It should also be used to introduce the concept of multiplication as repeated addition. It can be introduced during whole class teaching before children are given extensive opportunities to use it to work out questions independently.<br><br>By initially using Numicon to introduce the concept of multiplication it can help explain the role of the two numbers in a multiplication equation. So for 4 x 2 we pick up a two and multiply it four times. | Numicon<br>4 x 2 | U x 2<br><br>U x 5<br><br>U x 3 or 4 |
| **Things to remember** ||||
| 1) 4 x 2 means 4 lots of 2. While the order might not seem important because of the commutative law (the answer being the same regardless of the order) it makes it easier for the children if it is introduced in this way. ||||

# Method 2a: Labelled number line

| Explanation and Steps | Example | Progression of equations |
|---|---|---|
| **Labelled number line**<br><br>The labelled number line is the first written method that children should be taught. It can be used when children are still insecure with counting and the concept of multiplication.<br><br>Children start at 0 then count on in jumps of the second number. The first number shows how many jumps to make. The answer is the total after the jumps have been completed. | Labelled number line<br><br>2 x 3<br><br> | U x 2<br><br>U x 5<br><br>U x 3 or 4<br><br> |

<div align="center">

**Things to remember**

</div>

1. Numicon should still play a large part in teaching and learning as it helps to consolidate the children's conceptual understanding of multiplication.
2. 4 x 2 means 4 lots of 2. While the order might not seem important because of the commutative law (the answer being the same regardless of the order) it makes it easier for the children if it is introduced in this way.
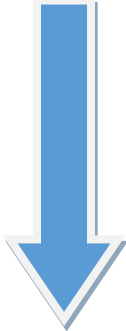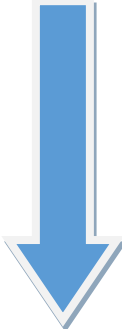3. At this stage children may need to count each jump in steps of 1. Therefore a step of 3 may require them to count three 1's to work out where the jump stops.
4. It is important that the children move to the empty number line as soon as they understand the concept of the number line **and** have the required numerical understanding.
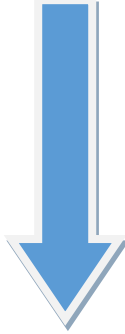
# Method 2b: Unlabelled number line

| Explanation and Steps | Example | Progression of equations |
|---|---|---|
| The unlabelled number line should be taught using the following steps with teachers only moving on when children have a secure understanding of that step:<br><br>**Step 1: U x U**<br><br>Children start at 0 then count on in jumps of the second number. The first number shows how many jumps they should make. The total after each jump should be marked down with the answer being the total after you have completed the jumps. | $\frac{U \times U}{5 \times 2}$<br><br>0 2 4 6 8 10 | U x 2<br><br>U x 5<br><br>U x 3 / 4 |

**Things to remember**

1. Numicon should still play a large part in teaching and learning as it helps to consolidate the children's conceptual understanding of multiplication.
2. 4 x 2 means 4 lots of 2. While the order might not seem important because of the commutative law (the answer being the same regardless of the order) it makes it easier for the children if it is introduced in this way.
3. The answer is the total after you have completed the jumps.

# Method 3: Informal jottings

| Explanation and Steps | Example | Progression of equations |
|---|---|---|
| Informal jottings should be taught to the most able children in Year 2. It is a bridge between the number line which should be used to work out U x U and the grid method which is introduced in Year 3.<br><br>**Step 1: TU x U**<br>Children start by partitioning the tens and units. They then multiply each of these numbers by the units before adding them up. This should primarily focus on teen numbers however the most able children can use it to multiply numbers between 20 and 30. | **TU x U**<br>16 x 5 = 80<br><br>16<br><br>10  +  6<br><br>↓    ↓  x5<br><br>50  +  30  =  80 | TU (under 20) x U<br><br>TU (20-30) x U |

**Things to remember**

1. Children need to be able to partition numbers into TU. You will therefore need to ensure that they can do this before attempting the method.
2. In Year 2 this method should, for the most part, only be used by potential level 3 children.
3. As a written method this will only be used by Year 2 children however it should be continue to be modelled and used as a mental method for and by children further up the school.

# Method 4: Grid Method

| Explanation and Steps | Example | Progression of equations |
|---|---|---|
| **Step 1: TU x U**<br><br>Children should estimate first before calculating the answer. This needs to be modelled and explained consistently.<br><br>Place the number that needs to be partitioned (H)TU in the left-hand column of the grid.<br><br>All numbers in grid should be lined up according to their place value, including the partitioned numbers in the left hand column. This will not impact on the answer but it provides another opportunity to reinforce this key skill. | **TU x U**<br>38 × 7 = 266<br><br>Estimate = 40 x 7 = 280<br><br><table><tr><td>x</td><td>7</td></tr><tr><td>30</td><td>210</td></tr><tr><td>8</td><td>56</td></tr><tr><td></td><td>266</td></tr></table> | TU (under 20) x U<br><br>TU x U<br><br>HTU x U |

In Year 5 children who are secure with this method need to move onto short multiplication (see method 5) while using the grid method to teach TU x TU and HTU by TU.

| Explanation and Steps | Example | Progression of equations |
|---|---|---|
| **Step 2: TU x TU**<br>Children should estimate first before calculating the answer. This needs to be modelled and explained consistently.<br><br>Both numbers should be partitioned with the larger number placed in the left-hand column of the grid.<br>The two products in the bottom of the grid should then be added next to the grid in a column.<br><br>All numbers in grid should be lined up according to their place value, including the partitioned numbers in the left hand column. This will not impact on the answer but it provides another opportunity to reinforce this key skill. | **TU x TU**<br>56 × 27<br><br>Estimate = 60 × 30 = 1800<br><br><table><tr><td>x</td><td>20</td><td>7</td></tr><tr><td>50</td><td>1000</td><td>350</td></tr><tr><td>6</td><td>120</td><td>42</td></tr><tr><td></td><td>1120</td><td>392</td></tr></table><br>1120<br>392<br>1512<br>1 | TU (under 20) x TU (under 20)<br><br>TU x TU |

| **Step 3: HTU x TU** | **HTU x TU** | |
|---|---|---|
| Children should estimate first before calculating the answer. This needs to be modelled and explained consistently. | 286 × 29 | HTU (between 100 and 200) x TU (under 20) |
| | Estimate = 300 × 30 = 9000. | |
| Both numbers should be partitioned with the HTU in the left-hand column of the grid so that it can be added vertically in two columns. | | |
| The two products in the bottom of the grid should then be added next to the grid in a column. | | HTU x TU (under 20) |
| All numbers in grid should be lined up according to their place value, including the partitioned numbers in the left hand column. This will not impact on the answer but it provides another opportunity to reinforce this key skill. | | HTU x TU |

| ×   | 20   | 9    |        |
|-----|------|------|--------|
| 200 | 4000 | 1800 | 5720   |
| 80  | 1600 | 720  | + 2574 |
| 6   | 120  | 54   | 8294   |
|     | 5720 | 2574 |        |

## Things to remember

1. All numbers in the grid should be lined up according to their place value.
2. Children should estimate first before calculating the answer. This needs to be modelled and explained consistently.
3. Both numbers should be partitioned with the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products. (If they have the same number of digits then partition the largest in the left hand column)
4. All children need to be using one digit per square in their books. If children are consistently doing this it makes lining up the digits far easier and helps avoid sloppy mistakes.
5. Use opportunities to reinforce place value through multiplication e.g. 30 x 6= 3 tens x 6 = 18 tens

# Method 5: Short Multiplication

| Explanation and Steps | Example | Progression of equations |
|---|---|---|
| **Step 1: TU x U and HTU x U**<br><br>Children should estimate first before calculating the answer. This needs to be modelled and explained consistently.<br><br>The recording from the grid method is reduced further, with digits being carried recorded below the line. See example opposite. Here children need to add 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage. | **TU x U**<br>38 x 7 = 266<br><br>Estimate =<br>40 × 7 = 280<br><br>$$\begin{array}{r} 38 \\ \times \quad 7 \\ \hline 266 \\ {\scriptstyle 5} \end{array}$$ | TU x U<br><br>HTU x U |
| **Step 2: Decimals**<br><br>Children should estimate first before calculating the answer. This needs to be modelled and explained consistently. Teachers will therefore need to ensure that children can round decimals.<br><br>Here children first need to multiply 0.7 x 7 writing the .9 in the answer box and placing the 4 underneath. They then need to multiply 4 x 7 before adding the 4 to the 28 and writing the 32 in the answer box. This highlights the need for children to be able to reliably use their knowledge of times tables and place value to multiply decimals. This needs to be practised consistently. | **U.t x U**<br>3.8 × 7 =<br><br>Estimate = 4 x 7 = 28<br><br>$$\begin{array}{r} 4.7 \\ \times \quad 7 \\ \hline 32.9 \\ {\scriptstyle 4} \end{array}$$ | U.t x U<br><br>U.th x U<br><br>*Questions can be extended beyond this for the most able students if required.* |

**Things to remember**

1) Children should estimate first before calculating the answer. This needs to be modelled and explained consistently.
2) All children need to be using one digit per square in their books. If children are consistently doing this it makes lining up the digits far easier and helps avoid sloppy mistakes.
3) Use opportunities to reinforce place value through multiplication e.g. 30 x 6= 3 tens x 6 = 18 tens
4) Only to be used in Year 5 and by children that can securely use the grid method.
5) Children need to be continually reminded of the value of the digits e.g. 3.8 × 7 = 3 units x 7 = 21 units.  8 tenths x 7 = 56 tenths. The children need to hear the value of the digits being referred to constantly to consolidate their understanding of place value.

# Written methods for division

While providing children with a strategy to answer division questions and problems the progression in methods below is vitally important for developing children's conceptual understanding of division without which children's mathematics cannot move beyond a basic level.
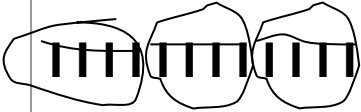
**NB:**

- It is important that children's **mental methods of calculation** and **number facts (times tables)** are practised and secured alongside their learning and use of an efficient written method for multiplication.
- Keep referring back to the big picture – multiplication is the inverse of division, multiplication is repeated addition (adding groups), division is repeated subtraction (subtracting groups)

## <u>Method 1: Introducing the concept of multiplication</u>

| Explanation and Steps | Example | Progression of equations |
|---|---|---|
| **<u>Cups</u>**<br>Introduce division using cups (idea from MMMS).<br>6 ÷ 2 = 3: Display 6 cups on a table.<br>Tell the children that the division sign means 'piles of'.<br>Make a pile of 2 cups, then make another pile of 2 etc.<br>Count the piles. How many piles of 2 cups are there? 3. | **<u>U ÷ 2</u>**<br>6 ÷ 2<br> | U ÷ 2<br><br>U ÷ 3<br><br>U ÷ 4<br> |

# Method 2a: Using informal methods to calculate division as grouping (repeated subtraction)

| Explanation and Steps | Example | Progression of equations |
|---|---|---|
| This method builds on the understanding that children have developed through exploring division with cups and provides children with an informal method that they can use to independently answer division questions.<br><br>**Step 1: U ÷ U**<br>Children place the dividend (the first number) into groups of the divisor (the second number) The quotient (the answer) is therefore the amount of groups. While modelling the method children's understanding should be visually and verbally supported using the cups' model. | **U ÷ U**<br>9 ÷ 3 = 3<br><br>*What to say*<br>*The first number tells us that we start with ninecups. The second number tells us that we need to put these into piles of 3. One group of three (draw around three), two groups of three, three groups of three. So our answer is 3.* | U ÷ 2<br><br>U ÷ 3<br><br>U ÷ 4 |
| **Step 2: TU ÷ U**<br>Children place the dividend (the first number) into groups of the divisor (the second number). The quotient (the answer) is therefore the amount of groups. While modelling the method children's understanding should be visually and verbally supported using the cups' model. | **Quickly move onto substituting cups for lines/circles**<br><br>**TU ÷ U**<br>12 ÷ 4 = 3 | TU (under 20) ÷ 2 / 3 / 4 / 5<br><br>TU (under 50) ÷ U |

**Things to remember**

1) Carefully model drawing the circle, making sure that children draw them in rows. Otherwise placing them in groups becomes far more difficult.
2) You also need to model how to group the circles, working from left to right and clearly separating the groups.
3) Sharing = Sharing 6 sweets between two children.   Grouping = Repeated subtraction (How many groups of two are there in 6)

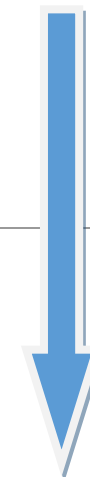# Method 2b: Using informal methods to calculate division as sharing (halving)

| Explanation and Steps | Example | Progression of equations |
|---|---|---|
| This method builds on the understanding that children have developed through exploring division with cups and provides children with an informal method that they can use to independently answer division questions (sharing).<br><br>**Step 1: U ÷ U**<br><br>Share the dividend (the first number) between the number of groups of the divisor (the second number) The quotient (the answer) is therefore the number in each group. While modelling the method children's understanding should be visually and verbally supported using the cups' model. | **U ÷ U**<br>8 ÷ 2 = 4<br><br><br><br><br>_What to say_<br>_The first number tells us that we start with eight cups. The ÷ sign can also mean share (as well as piles of).  The second number tells us that we need to share these equally between 2 groups.  We end up with two groups of four.  So our answer is 4._ | U ÷ 2<br><br>U ÷ 3<br><br>U ÷ 4 |
| **Step 2: TU ÷ U**<br><br>Children place the dividend (the first number) into groups of the divisor (the second number). The quotient (the answer) is therefore the amount of groups. While modelling the method children's understanding should be visually and verbally supported using the cups' model. | **Quickly move onto substituting cups for lines/circles**<br><br>**TU ÷ U**<br>12÷ 2 = 6<br><br>I<br><br><br>| | TU (under 20) ÷ 2 / 3 / 4 / 5<br><br>TU (under 50)  ÷ U |
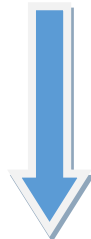
**Things to remember**

1) Children don't need to draw cups, a line or circle will suffice.

2) Carefully model drawing the circle, making sure that children draw them in rows. Otherwise placing them in groups becomes far more difficult.

3) You also need to model how to group the circles, working from left to right and clearly separating the groups.

4) Sharing = Sharing 6 sweets equally between two children.

   Grouping = Repeated subtraction (How many groups of two are there in 6)

# Method 3: Short division

| Explanation and Steps | Example | Progression of equations |
|---|---|---|
| **Step1 -** No carrying<br><br>The method works by working from the left dealing with all digits as if the numbers are units. The divisor is then divided by each digit with the answer put above the number. | TU ÷ U<br>88 ÷ 4 = 22<br><br>$$\begin{array}{c|c|c} & 2 & 2 \\ \hline 4 & 8 & 8 \end{array}$$ | TU ÷ U  – no carrying<br>HTU ÷ U  – no carrying |
| **Step 2 –** Remainders<br><br>As above but we are now dealing with remainders. These are put at the end of the answer. Children should also be taught that remainders can be expressed as a fraction of the divisor so in the example it could be r 2 or 2/3. | TU ÷ U with carrying<br>38 ÷ 3 = 12 r 2<br><br>$$\begin{array}{c|c|c} & 1 & 2 & r\ 2 \\ \hline 3 & 3 & 8 \end{array}$$ | TU ÷ U/HTU ÷ U  – with remainders |
| **Step 3 – C**arrying<br><br>As above but when there is a remainder this is then put next to the following number; making sure that you make the children explicitly aware that it should be considered as a ten. If no whole division can be made then a zero should be placed above it and the digit carried. | TU ÷ U with carrying<br>96 ÷ 8 = 12<br><br>$$\begin{array}{c|c|c} & 1 & 2 \\ \hline 8 & 9 & {}^{1}6 \end{array}$$ | Carrying no final remainders |

| | | |
|---|---|---|
| **Step 4** - Carrying and remainders<br><br>As above. Again children should be taught how to express remainders as fractions i.e. <u>remainder</u><br><div align="center">divisor</div><br>In the example opposite the answer would then be 12 <u>2</u><br><div align="center">8</div><br>With more able children knowing that this could then be expressed as 12 <u>1</u> or 12.25.<br><div align="left">   4</div> | <u>TU ÷ U with carrying and remainders</u><br>98 ÷ 8 = 12 r2<br><br>$$8 \overline{\smash{)}9^{1}8} \quad \begin{array}{c}1\,2 \,\,\,r\,2\end{array}$$ | Carrying and remainders |
| **Step 5** – Placing the quotient<br><br>This involves questions where the first digit of the dividend is less than the divisor. The first number of the quotient will then be zero.<br>Start off with simple questions with no carries before introducing questions including them. | <u>Noughts in the quotient middle digit</u><br>515 ÷ 5 = 103<br><br>$$5 \overline{\smash{)}5\,1\,5} \quad \begin{array}{c}1\,\,0\,\,3\end{array}$$ | Noughts in the quotient – final digit<br>Noughts in the quotient – middle digit<br>Decimals |

<div align="center">

**Things to remember**

</div>

1)  It is essential that at each stage the teacher explains what they are doing and that although the digit is a ten or a hundred, to help us we'll treat it as a unit.

2) While most people are familiar and confident with this method it is essential that the process is articulated consistently and accurately when modelled.

# Method 4: Long division

| Explanation and Steps | Example | Progression of equations |
|---|---|---|
| **Step 1**<br><br>In Year 5 and when children can confidently use the short division method to divide HTU by U then they can be shown how to divide HTU by TU using long division.<br><br>Children starting off by writing the multiples of the divisor down the right hand side of the page. They then note down how many of the divisor go into the first digit. This is 0 so they write this in the answer above and then find how many 12s go into 49. Here children need to use their list of multiples. This shows 4 lots of 12 makes 48 so we write 4 in our answer and subtract 48 from 49. This leaves us 1 so we now bring down the 2. We then know there is 1 lot of 12 in 12 so we write this in the answer.<br><br>The video below also helps to explain this, albeit using single-digit divisors.<br>http://www.youtube.com/watch?v=cvBZSgXSu38&feature=relmfu | Divisors of 11 and 12<br>492 ÷ 12 =<br><br>```
      0 4 1
12 | 4 9 2
      4 8
      ---
      1 2
      1 2
      ---
```<br>12<br>24<br>36<br>48<br>60<br>72<br>84<br>96<br>108<br>120 | Divisors of 11 and 12 no remainders<br>13 and 19 no remainders<br>Divisors between 21 and 29 no remainders<br>Divisors above 30 no remainders<br>Divisors below 30 with remainders<br>Divisors above 30 with remainders |

## Things to remember

1) This is the only way for children to divide HTU by TU as the short method is far too complex for dividing by TU.

2) The list of multiples down the right hand side is very important especially for more difficult questions.

3) Children need to be able to subtract using the column method in order to divide in this way.